

# Asymptotically Safe Gravitons in Electroweak Precision Physics

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Asymptotic safety offers a field theory based UV completion to gravity. For low Planck scales, gravitational effects on low-energy precision observables cannot be neglected. We compute the contribution to the  $\rho$  parameter from asymptotically safe gravitons and find that in contrast to effective theory, constraints on models with more than three extra dimensions are significantly weakened. The relative size of the trans-Planckian contribution increases proportional to the number of extra dimensions.

## I. INTRODUCTION

It is well known that despite the perturbative nonrenormalizability of general relativity, it is possible to compute gravitational quantum corrections with no assumptions other than those of effective theory [1]. However, it is also known that power-counting non-renormalizable theories may in some cases lead to quantum theories which are finite in the UV through non-perturbative dynamics. One well understood case is the Gross-Neveu model in  $2+1$  dimensions [2]. There is a growing amount of evidence that gravity is another theory featuring non-perturbative renormalizability or asymptotic safety [3–9]. The existence of fixed points in higher dimensional gravity [8], and their preservation upon coupling to matter [6], suggest that extra dimensional models with brane confined matter might also feature asymptotic safety. In this paper we will implement a framework for computing one-loop observables in the asymptotic safety scenario and compare with previous approaches from effective field theory. This follows a series of papers looking at calculable and measurable effects of asymptotically safe gravity in models with large extra dimensions [10], for example at the LHC [11–14].

Recent one-loop computations for the back reaction of gravity on the running of the gauge coupling have devoted a sizeable effort to the issue of gauge dependency in the final result [15]. Similarly, calculations for precision observables in low scale gravity/brane models have produced some debate on gauge fixing [16–24]. It was found that at the amplitude level these calculations produce a prohibitively large IR enhancement. These effects are gauge dependent and must disappear when calculated in proper observables [21]. Since the graviton couples universally and ubiquitously, the set of one-loop observables must be carefully quantified. However, our interest is not in the IR spectrum of gravity, and thus we will follow Ref. [21] and work exclusively in the DeDonder gauge where IR divergences are omitted from the outset. The physical interpretation of this gauge is that brane fluctuations decouple from the gravitational theory entirely and may be studied independently [25]. Using the DeDonder gauge we will compute corrections to the gauge boson propagators and find, as expected, that these are sensitive to the cutoff in both the momentum integral and Kaluza-Klein (KK) summation. At this point we will implement asymptotic safety in an attempt to quantify possible effects occurring at and above this scale.

It was argued previously that in extra dimensional models the physical scale identified with the renormalization group scale  $k$  should be the  $(4+n)$ -dimensional momentum of a KK graviton [14]. For the case of asymptotic safety this leads to finite, cut-off independent tree-level scattering amplitudes. We will demonstrate through explicit computation that one-loop corrections to gauge boson self energies are also finite in this set-up. Most importantly this means that we have no need to apply an explicit UV cutoff.

With a finite calculation for a well defined observable in our possession, we would like to explore two questions. First, how does the full computation in asymptotic safety compare with an effective theory approach. It is not clear *a priori* which will lead to stronger constraints as there are two competing effects. In effective theory the KK integral is cut-off separately in the KK mass  $m_{\text{KK}} < \Lambda$  and the graviton 4-

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momentum  $k < \Lambda$ , and thus (after an analytic continuation) may include modes with  $\sqrt{m_{\text{KK}}^2 + k^2} > \Lambda$ . The region outside this circle is suppressed by UV effects in our framework. On the other hand, modes with  $\max(k, m_{\text{KK}}) > \Lambda$  are not included in the effective theory computation, while these modes will contribute in our computation albeit with the UV suppression provided by asymptotic safety. The results may then also be compared with constraints on extra dimensions from collider [26], astrophysical [27], and precision tests of gravity [28].

Second, we would like to have some sense of the ratio of UV to IR contributions as a function of the number of extra dimensions. We know that tree-level amplitudes can become UV dominated for 6 and more extra dimensions [12]. In this paper we will show that at one-loop UV domination may begin at lower dimensions.

## II. SELF ENERGY

Our task in this section is, starting from the conventional picture of large extra dimensions, to compute the self energy amplitudes using standard perturbative techniques, and thus reduce the general tensor integral into a closed set of scalar integrals. We will restrict the brane dimension to strictly  $(3+1)$ , *i.e.* we will not use dimensional regularization. Thus, we consider a gravitational theory in  $(4+n)$  space-time dimensions with the coupling to matter

$$\frac{32\pi}{M_*^{2+n}}, \quad (1)$$

where  $M_*$  is the fundamental scale of gravity. The normalization factor matches the notation of Ref. [29]. For a realistic model we require some mechanism which generates the observed Planck mass and generically this will involve a volume factor  $V_n$  of the extra dimensions. Thus, in the case of a torus compactification we have the for observed 4-dimensional gravitational coupling  $M_{\text{Pl}}^2 = (2\pi R)^n M_*^{2+n}$ . The utility of these conventions is that the integral and KK sum associated with each graviton loop is

$$\frac{1}{M_{\text{Pl}}^2} \sum_{\text{KK}} \int \frac{d^4 k}{(2\pi)^4} = \frac{1}{M_*^{2+n}} \int \frac{d^{4+n} q}{(2\pi)^{4+n}}, \quad (2)$$

which is merely a loop integral over the  $(4+n)$ -dimensional momentum. The sum is shorthand for the individual sums over the KK occupation numbers. Later, we will need the 4-dimensional brane projection of  $q$ , which we denote as  $k$ .

The gauge fixing and KK reduction is standard [29]. On the brane we have simply KK gravitons and KK scalars in the spectrum, minimally coupled to the Standard Model. In the KK reduction, one also in general obtains KK vector fields, which do not couple to matter fields at this order in the expansion. The graviton and scalar propagators are required for our computations and are derived by inverting the kinetic terms of the Einstein-Hilbert action. In the classical regime we expect the scalar part for both to be  $\Delta_G = (k^2 - m_{\text{KK}}^2 + i\epsilon)^{-1}$ , but at high energies we anticipate different behavior.

The vertices coupling the scalar and graviton to the brane confined SM can be obtained by expanding the metric in the massive gauge boson action

$$g_{\mu\nu} = \sum_{\vec{n}} \eta_{\mu\nu} \left( 1 + \frac{16\pi}{M_{\text{Pl}}} \phi^{(\vec{n})} \right) + \frac{32\pi}{M_{\text{Pl}}} h_{\mu\nu}^{(\vec{n})} + \dots \quad (3)$$

The sum is over the discrete momenta in the theory. Implicitly in this step we have assumed a compact space and enforced periodic conditions. The ellipses stand for terms, in addition to higher order in  $M_{\text{Pl}}^{-1}$ , from the vector KK modes and brane fluctuations (branons) which do not contribute in our computations [21, 29].

We start with the seagull diagram,

$$\begin{aligned}
 \Pi_S(p^2) &= \frac{1}{2} \text{ (seagull diagram) } \\
 &= -\frac{32\pi}{M_*^{2+n}} \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G \frac{3}{2} (p^2 \eta^{\mu\nu} - p^\mu p^\nu),
 \end{aligned} \tag{4}$$

in agreement with the amplitude for a massless gauge boson. Here,  $p$  is the external momentum,  $q$  is the bulk loop momentum, and  $\Delta_G$  the scalar part of the graviton propagator. Furthermore, the gravi-scalar contribution in four dimensions is proportional to the trace of the energy-momentum tensor and thus can only couple to the mass term. However we find in the metric expansion that there is no term of order  $M_{\text{Pl}}^{-2}\phi^2$  and thus the only contribution from extra dimensions is Eq.(4). We will not simplify the loop integral at this point. The rainbow diagram is

$$\begin{aligned}
 \Pi_R(p^2) &= \text{ (rainbow diagram) } + \text{ (rainbow diagram) } \\
 &= \frac{32\pi}{M_*^{2+n}} \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G \frac{A^{\mu\nu}(k, p)}{(k+p)^2 - m_V^2},
 \end{aligned} \tag{5}$$

where  $A_{\mu\nu}$  contains additionally terms proportional to  $m_V^2$  and  $m_V^4$ . The exact form is given in the Appendix A. We have also introduced  $k$ , the brane only component of the full loop momentum. At this point we could introduce Feynman parameters and perform a cut-off or dimensional regularization. We expect that the leading UV divergences would cancel with similar terms from the seagull diagram. However, the amplitude would retain power and logarithmic sensitivity to the cut-off scale [21]. In both cases the amplitude is UV sensitive, and the effects of quantum gravity are explicitly cut-off.

As noted in the introduction, the fixed point scaling which we will employ leads to a finite amplitude. Therefore, we will tensor reduce this amplitude without making specific reference to the regulator. Additionally, the amplitude in Eq.(5) can be reduced to scalar integrals without any knowledge of the higher dimensional theory. Using Lorentz invariance on the brane, we project the tensor integral onto the scalar integrals

$$\begin{aligned}
 \tilde{A}_0 &= \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G \\
 \tilde{B}_0 &= \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G \frac{1}{(k+p)^2 - m_V^2} \\
 p^2 \tilde{B}_1 &= \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G \frac{p \cdot k}{(k+p)^2 - m_V^2} \\
 p^2 \tilde{B}_2 &= \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G \frac{k^2}{(k+p)^2 - m_V^2}
 \end{aligned} \tag{6}$$

The notation is meant to evoke the similarity to the normal basis integrals obtained in Passarino-Veltman reduction [31]. The seagull contribution simply reads

$$\Pi_S(p^2) = -\frac{32\pi}{M_*^{2+n}} \frac{3}{2} (p^2 \eta^{\mu\nu} - p^\mu p^\nu) \tilde{A}_0, \tag{7}$$

while the rainbow self energy in terms of the basis integrals is

$$\begin{aligned} \Pi_R(p^2) = & \frac{16\pi}{M_*^{2+n}} \times \left( (p^2 \eta^{\mu\nu} - p^\mu p^\nu) \left[ 12m_V^2 (\tilde{B}_1 + \tilde{B}_0) \right. \right. \\ & \left. \left. + 3p^2 \tilde{B}_2 + 16p^2 \tilde{B}_1 + 8p^2 \tilde{B}_0 \right] + \eta^{\mu\nu} m_V^4 \left[ 10 + \frac{n}{2+n} \right] \tilde{B}_0 \right). \end{aligned} \quad (8)$$

It should be noted that we have not taken into account brane fluctuations, coupling proportional to the brane tension  $\tau$ . It is not clear how one would incorporate this parameter within the framework of asymptotic safety. Moreover, we are physically justified to neglect this contribution if we restrict ourselves to rigid branes or more precisely the energy hierarchy described in Ref. [29].

### III. MOMENTUM INTEGRATION

Quantum corrections to a massless propagator can be simply accounted for by the wave function renormalization  $Z(\mu)$ . In particular, we define the renormalization group improved scalar propagator

$$\Delta(q) = \frac{1}{Z} \frac{1}{q^2}, \quad (9)$$

for a massless graviton. The specific form of  $Z^{-1}$  encodes, in general in a non-perturbative way, the quantum effects in the theory. For practical purposes we may only calculate the fixed point behavior of  $Z$  in certain approximations, *e.g.* with a truncated action or at fixed order in perturbation theory. For this study we will use three approximations for  $Z^{-1}$  motivated by asymptotic safety. First, we define the linear [14]

$$\Delta^{\text{linear}} = \frac{1}{q^2} \left[ 1 + \frac{\mu^{n+2}}{\Lambda_T^{n+2}} \right]^{-1}, \quad (10)$$

and quadratic approximation

$$\Delta^{\text{quadratic}} = \frac{1}{q^2} \left[ \sqrt{1 + \left( \frac{\mu^{n+2}}{2\Lambda_T^{n+2}} \right)^2} - \frac{\mu^{n+2}}{2\Lambda_T^{n+2}} \right]. \quad (11)$$

The numerical value of  $\Lambda_T$  is numerically related to  $g^*$ , the fixed point value of the dimensionless coupling, which itself carries gauge and cut-off dependency. Therefore, for our purposes  $\Lambda_T$  is treated as input although related to the fundamental scale  $M_*$  by a parameter of  $\mathcal{O}(1)$ . The factor two in Eq.(11) is convenient when matching with the one-loop perturbative result [1], and thus is left explicit rather than being absorbed into the scale  $\Lambda_T$ .

As opposed to the quenched approximation [7]

$$\Delta^{\text{quenched}} = \begin{cases} \frac{1}{q^2} & q^2 < \Lambda_T^2 \\ \frac{1}{q^2} \frac{\Lambda_T^{2+n}}{|q|^{2+n}} & q^2 > \Lambda_T^2 \end{cases}, \quad (12)$$

the graviton propagator in (10) and (11) does not contain additional poles and thus has only the simple poles corresponding to the classical propagator. These modified propagators falls off with sufficient power suppression for large values of  $q_0$ , so the contour integral can be Wick rotated as usual.

Finally, we note that recent studies indicate that the running of the gauge couplings induced by the gravitational coupling vanish once a consistent regulator is chosen which respects the appropriate symmetries [24]. Therefore, while the graviton propagator is strongly altered above  $\Lambda_T$ , we assume that the gauge boson propagator remains classical.

### A. Finiteness

We can heuristically show the finiteness of our calculation in the asymptotic safety framework. We apply both the quadratic (11) and quenched (12) approximations to account for the large anomalous dimension in the UV. First of all, we consider the UV portion of the seagull integral in Eq.(4), assuming the wave-function renormalization in the the quenched approximation Eq.(12). The momentum integral is then trivial

$$\Pi_S^{\text{quenched}} \sim \int_{\Lambda_T}^{\Lambda_{UV}} \frac{dq}{q} = \log \frac{\Lambda_{UV}}{\Lambda_T} . \quad (13)$$

For the rainbow diagram we first write the brane momenta as the 4-dimensional projection of the  $(4+n)$ -dimensional momentum *i.e.*  $k^2 = q^2 \chi^2$ . We will only display an explicit form of  $\chi$  at a later point, since it is not required for showing UV finiteness. Considering small values of the external momenta, the leading divergence is

$$\Pi_R^{\text{quenched}} \sim \int_{\Lambda_T}^{\Lambda_{UV}} \frac{dq}{q} \frac{q^2 \chi^2}{q^2 \chi^2 + m_V^2} = \log \frac{\Lambda_{UV}}{\Lambda_T + \dots} . \quad (14)$$

Once we properly perform the tensor reduction the seagull and rainbow diagrams acquire pre-factors  $(3/2)(p^2 \eta^{\mu\nu} - p^\mu p^\nu)$  with opposite signs, and the sum of Eq.(13) and Eq.(14) becomes independent of  $\Lambda_{UV}$  and hence finite.

It is instructive as well to consider the same integrals for the quadratic approximation. For the seagull we find

$$\Pi_S^{\text{quadratic}} \sim \frac{\Lambda_T^{n+2}}{(n+2)} \sinh^{-1} \frac{\Lambda_{UV}^{n+2}}{2\Lambda_T^{n+2}} . \quad (15)$$

We obtain the term in  $\Lambda_{UV}$  by expanding Eq.(15) and find the same leading logarithmic dependence on  $\Lambda_{UV}$  as in the quenched case. The rainbow we evaluate using the techniques described above, and when summed with Eq.(15) the result is again finite. Had either of these computations provided sub-leading divergences of any type, we would not have a sufficiently regulated theory, so the fact that the only divergences are logarithmic serves as evidence of finiteness at one-loop.

We have not mentioned terms in the amplitude proportional to the  $m_V^2$  and  $m_V^4$ . By power counting the later cannot produce a divergence, while the former can at worst admit a term proportional to  $p \cdot k$  and not  $k^2$ . This can be seen by examining the momentum structure of the vertices. These terms are individually finite in asymptotically safe gravity.

The lack of sensitivity to  $\Lambda_{UV}$  in the self energy amplitude is of course only valid for the DeDonder gauge and will not be true for any other choice. However, also in a less appropriate gauge any physical observable must be independent of  $\Lambda_{UV}$ . To emphasize this point we outline the related computation in unitary gauge. The amplitude for the seagull diagram is

$$\begin{aligned} \Pi_S^{\text{unitary}} = \frac{32\pi}{M_*^{2+n}} \int \frac{d^{4+n}q}{(2\pi)^{4+n}} \Delta_G (p^2 \eta^{\mu\nu} - p^\mu p^\nu) \times \\ \left( \frac{3}{2} - \frac{3}{4} \frac{k^2}{k_T^2} + \frac{1}{16} \frac{k^4}{k_T^4} \right) , \end{aligned} \quad (16)$$

with a similar, albeit lengthy, expression for the rainbow. There are no higher UV divergences since upon substitution as described in the following sections the additional terms are proportional only to the ratio of the angular parts, with no  $q$  dependence. However, the cancellation of the additional logarithmic terms must also be checked. For genuine physical observables, such as the one presented later in Section IV, this cancellation does indeed take place and the result is finite.

### B. Warm-up in 2 + 1 dimensions

To illustrate the geometrical picture of our loop integral we consider the same integral with a (Euclidean) 2 dimensional brane and a single extra dimension. In momentum space we can easily picture the integral over brane and bulk momenta as a three dimensional integral in terms of spherical polar coordinates. For a rainbow like diagram we have

$$\begin{aligned}\Pi_{2+1}(p^2) &= \int d^3q \frac{1}{q^2} \frac{k_2^2}{(k_2 + p)^2 + m_V^2} \\ &= \int dq q^2 \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi \\ &\quad \frac{1}{q^2 \sin^2 \theta + 2|p||q| \sin \theta \cos \phi + p^2 + m_V^2},\end{aligned}\tag{17}$$

where we denote the 2-dimensional brane analogy of the full momentum by  $k_2$ . The spherical coordinates we orient such that  $p$  is located at  $\phi = 0$  and the dot product occurs between the brane projection  $|q| \sin \theta$  with the brane external momentum  $p$ . This integral is evaluated numerically, or in this simple case analytically over the angular coordinates. The full result in  $(4 + n)$  dimensions is merely a higher dimensional generalization of Eq.(17).

### C. Result in 4 + n dimensions

Having established the finiteness of our computation in both the quenched and quadratic approximation, we can numerically evaluate the rainbow and seagull diagrams. It is clear at this point that we would like to perform the  $(4 + n)$ -dimensional momentum integral over a radial coordinate  $|q|$ , but the loop momentum in the gauge boson propagator depends on only the brane projection of  $|q|$ . Thus our integral in  $(4 + n)$  dimensions must retain some angular dependence as the bulk and brane momentum are not interchangeable in the rainbow diagram.

A straightforward solution is to define a  $(4 + n)$  dimensional Euclidean vector  $q = (k_T, k_4)$  in polar coordinates. Treating the brane momentum as the last entries in this  $(4 + n)$ -vector allows us to make the variable change  $k_4^2 + k_T^2 \equiv q^2$  which in turn requires the projection

$$k_4^2 \equiv q^2 \sin^2 \phi_1 \sin^2 \phi_2 \cdots \sin^2 \phi_n \equiv q^2 \chi^2, \tag{18}$$

where the last step is simply a shorthand notation. The physical interpretation of the angular variables should be clear as they measure the 4-dimensional projection of  $q$ . The configuration with all  $\phi_i = \pi/2$  corresponds to momenta confined to the brane, and the corresponding integral is the conventional 4-dimensional case. This simplifies our loop integrals and is easily evaluated numerically. Under the coordinate change the measure in our  $(4 + n)$ -dimensional loop integration should be replaced as

$$\begin{aligned}\int \frac{d^{4+n}q}{(2\pi)^{4+n}} &= \frac{\pi}{(2\pi)^{4+n}} \int dq q^{3+n} \int_0^\pi d\phi_1 \sin^{2+n} \phi_1 \cdots \\ &\quad \int_0^\pi d\phi_n \sin^3 \phi_n \int_0^{2\pi} d\phi_{3+n}.\end{aligned}\tag{19}$$

The pre-factor  $\pi$  in front is the result of integrating over the 2-additional brane angular coordinates  $\phi_{1+n}$  and  $\phi_{2+n}$  which our amplitude does not depend on. For the angular measure related to  $\chi$  we write  $\int d\chi$ , so the basis integrals from Eq.(6) with the renormalization group improved scalar propagator in Eq.(9) become

$$\tilde{A}_0 = \frac{2}{(2\pi)^{4+n}} \frac{\pi^{n/2+2}}{\Gamma[n/2+2]} \int dq q^{n+1} Z^{-1} \tag{20}$$

$$\tilde{B}_0 = \frac{\pi}{(2\pi)^{4+n}} \int dq q^{3+n} \int d\chi \int_0^{2\pi} d\phi_{3+n} \frac{Z^{-1}}{q^2} \frac{1}{q^2 \chi^2 + 2|p||q| \cos \phi_{3+n} \chi + (p^2 + m_V^2)} \quad (21)$$

$$p^2 \tilde{B}_1 = \frac{\pi}{(2\pi)^{4+n}} \int dq q^{3+n} \int d\chi \int_0^{2\pi} d\phi_{3+n} \frac{Z^{-1}}{q^2} \frac{|p||q| \cos \phi_{3+n}}{q^2 \chi + 2|p||q| \cos \phi_{3+n} + (p^2 + m_V^2) \chi^{-1}} \quad (22)$$

$$p^2 \tilde{B}_2 = \frac{\pi}{(2\pi)^{4+n}} \int dq q^{3+n} \int d\chi \int_0^{2\pi} d\phi_{3+n} \frac{Z^{-1}}{q^2 + 2|p||q| \cos \phi_{3+n} \chi^{-1} + (p^2 + m_V^2) \chi^{-2}} \quad (23)$$

Using these basis integrals we can numerically compute the gauge bosons self energies in Eq.(4) and Eq.(8). The sum of the two integrals does not depend on  $\Lambda_{UV}$ . In the next section we will see how this method of computing the KK integral compares with an effective theory analysis.

#### IV. RESULTS

In order to constrain new physics based on precision measurements it is typical to use either the oblique parameter set  $\{S, T, U\}$  [32] or the  $\epsilon$  parameterization [33]. However, the former is not a good parameter set for gravitons, which in general will modify more than just gauge boson self-energies. The later can be related to the weak mixing angle  $s_0, c_0$  to define the leading corrections

$$\rho - 1 \simeq \bar{\epsilon} = \epsilon_1 - \epsilon_2 - \frac{s_0^2}{c_0^2} \epsilon_3, \quad (24)$$

quantifying the violation of custodial symmetry [34, 35]. The individual observables  $\epsilon_1, \epsilon_2$  and  $\epsilon_3$  in the context of gravity require additional computations beyond the self-energies, but the specific linear combination

$$\bar{\epsilon} = \frac{\Pi(m_W^2)}{m_W^2} - \frac{\Pi(m_Z^2)}{m_Z^2} \quad (25)$$

has to be smaller than roughly  $10^{-3}$ , assuming a light Standard Model Higgs boson [36]. The exact central value depends on the Higgs mass, but the uncertainty of  $\bar{\epsilon}$  also ranges around  $10^{-3}$ , which makes this value a generic upper bound on new physics effects. The effective theory result for gravity contributions [21] in our notation reads

$$\Delta\rho \simeq \Delta\bar{\epsilon} = \frac{s^2 m_Z^2}{M_*^2} \left( \frac{\Lambda_{\text{eff}}}{M_*} \right)^n \frac{1}{\Gamma(2+n/2)} \frac{5(8+5n)}{48\pi^{2-n/2}}, \quad (26)$$

for  $m_{KK} \gg m_Z$ . Here,  $\Lambda_{\text{eff}}$  is the effective theory cut-off scale. In Figure 1 we compare this result to our computation as described in the previous section. The numerical results are similar for  $n = 3$ , where fundamental Planck masses below  $M_* \lesssim 1$  TeV are forbidden. In asymptotically safe gravity the limits are significantly weakened when we increase the number of extra dimensions, which is not the case for the effective theory computation in Eq. (26). In other words, once we take the higher dimensional quantum effects seriously there are essentially no limits on large extra dimensions with  $n > 3$  from electroweak precision data.

The explanation for this discrepancy is the following: in effective theory, the amplitudes are dominated by modes with both  $k$  and  $m_{KK}$  near the cut-off, *i.e.* in the corner of a square in the  $k$  vs  $m_{KK}$

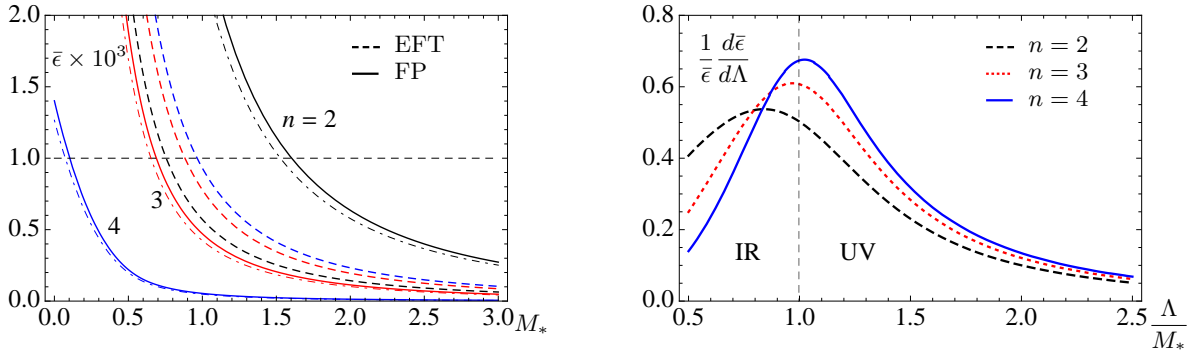


FIG. 1: Left: contribution to  $\bar{\epsilon} \simeq 1 - \rho$  from extra dimensions computed in asymptotically safe gravity in linear (10) (dot-dashed), quadratic (11) (solid) and in effective theory (dashed) [21]. Values above the dashed horizontal curve are in tension with data. For all asymptotic safety curves we have taken  $M_* = \Lambda_T$  and in the EFT case we have  $M_* = \Lambda_{\text{eff}}$ . Right: contribution to  $\bar{\epsilon}$  as a function of the highest momentum mode  $q < k_0$  in the  $(4 + n)$ -dimensional integration. We see the approximate pattern of Eq. (27) in the ratio of IR to UV contributions.

plane. However, in our picture of asymptotic safety with  $|\mu| = \sqrt{k^2 + m_{\text{KK}}^2}$  this region is outside the central circle which means it is suppressed. For higher numbers of extra dimensions this effect is more pronounced, as shown by the lessening contribution to  $\bar{\epsilon}$  as  $n$  increases.

To confirm our observation of an increasing impact from the UV regime we need to study the distribution of the momentum modes contributing to  $\bar{\epsilon}$ . More specifically, we would like to know the size of the contribution to the observable for UV momentum modes with  $|q| > \Lambda_T$ . In the quenched approximation we can estimate this fraction from Eq.(12). Neglecting terms of order  $m_V/\Lambda_T$  the leading momentum integrals are of the form

$$\begin{aligned} \bar{\epsilon} \Big|_{\text{IR}} &\sim \int_0^{\Lambda_T} dq q^{3+n} \frac{1}{q^2} \frac{1}{q^2 + \dots} \approx \frac{\Lambda_T^n}{n} \\ \bar{\epsilon} \Big|_{\text{UV}} &\sim \int_{\Lambda_T}^{\infty} dq q^{3+n} \frac{\Lambda_T^{2+n}}{q^{4+n}} \frac{1}{q^2 + \dots} \approx \frac{\Lambda_T^n}{2}. \end{aligned} \quad (27)$$

This expansion in  $m_V/\Lambda_T$  is well validated, for example compared with LHC tree level virtual graviton exchange where  $\sqrt{s}/\Lambda_T$  can easily become  $\mathcal{O}(1)$  [11]. For  $n = 2$  approximately 50% of the combined integral comes from the quantum gravity regime, and for a larger number of extra dimensions this fraction increases. Numerically, Figure ?? shows that the exact results follow the pattern delineated in Eq.(27). The UV region becomes the dominant contribution already for  $n > 2$  extra dimensions.

This can be contrasted with the case of LHC tree level graviton exchange, where in an identical limit the UV graviton contribution becomes dominant for  $n > 6$  [12]. This is not surprising though as the momentum integral in the tree-level exchange is over four less directions in momentum space, and thus we see the rough equivalence of the  $n = 6$  case in tree-level with the  $n = 2$  case at one-loop.

## V. CONCLUSION

Asymptotically safe gravity allows us to compute quantum gravitational effects on relevant observables. The ultraviolet or quantum gravity regime does not require any modified treatment or cut-off procedure, *e.g.* to ensure finiteness of our predictions. In models with large extra dimensions [10] gravitational effects should be measurable, since the fundamental Planck scale does not lead to a significant suppression, as compared to other TeV scale physics or electroweak loop effects. In this framework, asymptotic safety is particularly useful because it provides measurable predictions for the LHC [11] or, as we have shown in the work, for electroweak precision measurements.

We first introduced a method for evaluating gravitational loop integrals in extra-dimensions. Defining these in terms of the full  $(4 + n)$ -dimensional momentum provided a straight-forward regularization



scheme. Similar to the usual Passarino-Veltman reduction into scalar one-loop integrals, we defined and numerically evaluated a set of basis integrals which can be used for a wide class of observables. In this paper we studied gravitational effects on custodial symmetry in the Standard Model, *i.e.* the  $\rho$  parameter or  $\bar{\epsilon}$ .

We find that the bounds from electroweak precision data based on asymptotic safety are roughly equivalent to the direct/indirect bounds obtained by previous methods for  $n = 3$  extra dimensions. For higher numbers of extra dimensions the bounds from virtual gravitons are irrelevant compared with more direct measurements. Compared to an effective field theory (or cut-off) prescription our limits are weaker for more than two extra dimensions. This is consistent with the general observation that for larger numbers of extra dimensions the relative contribution from the trans-Planckian (and hence strongly suppressed) regime become more and more dominant. No matter what kind of ultraviolet completion of gravity should be chosen by Nature, this paper shows that it has to be modeled properly and quantitatively taken into account.

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### Appendix A: Amplitude

In Eq.(5) we defined  $A_{\mu\nu}$  which is given here for convenience.

$$A^{\mu\nu}(k, p) = A_1^{\mu\nu}(k, p) + m_V^2 A_2^{\mu\nu}(k, p) + m_V^4 A_3^{\mu\nu} \quad (\text{A1})$$

$$\begin{aligned} A_1^{\mu\nu}(k, p) = \frac{1}{2} [ & -4p^\mu p^\nu p^2 - 4p^\mu p^\nu p \cdot k - p^\mu p^\nu k^2 \\ & - 4p^\mu k^\nu p^2 - 3p^\mu k^\nu p \cdot k + p^\nu k^\mu p \cdot k \\ & - k^\mu k^\nu p^2 + 8\eta^{\mu\nu} p^2 p \cdot k + \eta^{\mu\nu} p^2 k^2 \\ & + 4\eta^{\mu\nu} p^4 + 3\eta^{\mu\nu} (p \cdot k)^2 ] \end{aligned} \quad (\text{A2})$$

$$A_2^{\mu\nu}(k, p) = 3(\eta^{\mu\nu}(p \cdot k + p^2) - p^\mu k^\nu - p^\nu p^\mu) \quad (\text{A3})$$

$$A_3^{\mu\nu} = \frac{5}{2} \eta^{\mu\nu} \quad (\text{A4})$$

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